Extreme Value Theory and Fat Tails in Equity Markets

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Abstract

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Equity market crashes or booms are extreme realizations of the underlying return distribution. This paper questions whether booms are more or less likely than crashes and whether emerging markets crash more frequently than developed equity markets. We apply Extreme Value Theory (EVT) to construct statistical tests of both of these questions. EVT elegantly frames the problem of extreme events in the context of the limiting distributions of sample maxima and minima. This paper applies generalized extreme value theory to understand the probability of extreme events and estimate the level of "fatness" in the tails of emerging and developed markets.

We disentangle the major "tail index" estimators in the literature and evaluate their small sample properties and sensitivities to the number of extreme observations. We choose to use the Hill index to measure the shape of the distribution in the tail. We then apply nonparametric techniques to assess the significance of differences in tail thickness between the positive and negative tails of a given market and in the tail behavior of the developed and emerging region. We construct Monte Carlo and Wild Bootstrap tests of the null of tail symmetry and find that negative tails are statistically significantly fatter than positive tails for a subset of markets in both regions. We frame group bootstrap tests of universal tail behavior for each region and show that the tail index is statistically similar across countries within the same region. This allows us to pool returns and estimate region wide tail behavior. We form bootstrapping tests of pooled returns and document evidence that emerging markets have fatter negative tails than the developed region. Our findings are consistent with prevalent notions of crashes being more in the emerging region than among developed markets. However our results of asymmetry in several markets in both regions, suggest that the risk of market crashes varies significantly within the region. This has important implications for any international portfolio allocation decisions made with a regional view.

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Introduction

The exact distribution of financial returns remains an open question. Stylized facts about the distribution of financial returns generally concur that at monthly and longer horizons, return series appear to be normally distributed. However at weekly, daily and higher frequencies, return distributions consistently display non-normal features. One such characteristic that arises across financial assets, from foreign exchange returns (De Vries,1991) to commodities such as cotton or wheat (Mandelbrot, 1963) is “fat tails”. Fat tails are defined as tails of the distribution that have a higher density than that what is predicted under the assumption of normality. For example, a distribution that has an exponential decay (as in the normal) or a finite endpoint is considered thin tailed, while a power decay of the density function in the tails is considered a fat tailed distribution.

This paper draws on extreme value theory to frame the question of fat tails and study how tail characteristics vary between markets and regions. Fat tails are consistent with a variety of financial theories. Herding theories rely on asymmetric or noisy information to create cascades in particular directions resulting in the clustering of extreme events. They are also consistent with theories of asymmetric response to news. For instance, good news will raise prices but also increase price volatility, raising risk premia and dampening the market rise. On the other hand, with bad news, the decline in prices is met with a rise in volatility causing further decline in the same direction. This might explain the common perception that busts are more probable and extreme than market booms.

This study poses the following questions. Firstly are return distributions asymmetric? Recent behavioral finance literature abounds with evidence of loss aversion and nonlinear perception of risk as investors typically perceive crashes as being more painful and probable than booms. We aim to see if the differences between extreme positive and negative realizations of the return distribution are statistically demonstrable. Secondly are fat tails more prevalent in emerging equity markets? Evidence of this would be consistent with the common view that emerging markets are less efficient than developed markets.

Section I opens with a discussion of extreme value theory and its implications for our study of fat tails. Section II reviews, estimates and compares a range of tail index measures that are possible candidates for characterizing tail behavior. Given these results, Section III applies the Hill index to estimate and compare tail behavior across tails through monte carlo and bootstrapping methods. Section IV applies monte carlo techniques to compare tail features between markets in developed and emerging regions. Finally Section V draws together conclusions from the various tests and discusses their implications for risk management.

I. Extreme Value Theory

Extreme value theory provides a framework to formalize the study of behavior in the tails of a distribution. Critical questions relating to the probability of a market crash or boom require an understanding of the statistical behavior expected in the tails. EVT allows us to use extreme observations to measure the density in the tail. This measure can be extrapolated to parts of the distribution that have yet to be observed in the empirical data. It can also be mapped onto distributions with specific tail behavior. In this way we can simulate a theoretical process that captures the extreme features of the empirical data and improve the accuracy of estimated probabilities of extraordinary market movements.

According to the central limit theorem, the normal distribution is the limiting distribution of sample averages. A parallel idea exists when discussing sample extrema. A class of extreme value distributions characterize the possible distributions of sample maxima. The Fisher-Tippet theorem is analogous to the CLT and uses the tail index to unify the possible characterizations of the density function of an extreme value distribution. Intuitively these functions represent three possibilities for the decay of the density function in the tail. These three possibilities can be mapped onto different domains of attraction, the Gumbel, Frechet and Weibull limiting distributions.

a) The tail can decay exponentially with all finite moments; these are the standard cases of the normal, log normal, gamma etc .
b) It can decay by a power, as in the cases of the stable Paretian, Cauchy and student t distributions. These are no longer integrable when weighed by the tail probabilities, hence leading to " fat tails".
c) The tail can decay with a finite tail index; this will be a thin tailed distribution with a finite upper endpoint.

Suppose X_1, X_2, X_3 are a sequence of stationary random variables that may be either iid or dependent with a common distribution function $F(x) = \Pr\{X_t < x\}$. For the first set of n observations, we get the maxima M_1 . The next set of n observations of X_2 yields the next maximum M_2. In this way from n*N observations one would get N observations of the maxima. Extreme Value Theory tells us that even without exact knowledge of the distribution of the parent variable (X) we can derive certain limiting results of the distribution of M. In order to do this, M is first reduced by a location parameter (υ) and a scale parameter (σ). This standardized extreme variable has a distribution that is non-degenerate. As the period (n) over which the maximum is taken tends toward infinity, the Fisher Tippet theorem summarizes three possible limiting extreme value distributions for this standardized variable.

Formally, one is interested in the conditions under which there exist normalizing constants $a_n > 0, b_n$ such that the distribution of the standardized block maxima (M_n) weakly follows a specific type of extreme value distribution function G. These three types of distributions map onto the tail possibilities described in (a-c) above.

Definition 1: A nondegenerate distribution function G is called max-stable if there exist real constants $a_n > 0, b_n$ such that for all real x and n=1,2,….
$G^n(a_n x + b_n) = G(x)$
Then G(x) is of one of the following three forms:

Type I:
$$G(x) = \exp(-e^{-x}) \rightarrow -\infty < x < \infty$$

Type II:
$$G(x) = 0 \rightarrow x <= 0$$
$$G(x) = \exp(-x^{-\alpha}) \rightarrow x > 0$$

Type III:
$$G(x) = \exp(-(-x)^{\alpha}) \rightarrow x < 0$$
$$G(x) = 1 \rightarrow x >= 0 \qquad \text{(Equation 1)}$$

The unifying feature across these distributions is the shape parameter (α). This captures the weights of the tail in the distribution of the parent variable (X). In all cases the scale, location and shape parameters – here denoted as a, b and α may vary between the negative and positive tails.

Since we are concerned here with stock market returns that are known to be fat tailed, we can rule out the possibility of a type I distribution. Since returns are theoretically unbounded, we can also exclude the type III limiting distribution. Thus we can focus on the Frechet domain of attraction which encompasses numerous distributions ranging from the student-t to the stable. In this domain, α is related to the maximal order moment. This means that moments of order r greater than (α) are infinite and those less than (α) are finite and well-defined. Hence the student t distribution has a shape parameter α>= 2 and the stable distribution has a tail parameter $0< \alpha <2$. The student t has a well defined mean and variance, while the stable has a mean but not a finite variance This further points to the importance of the accuracy of the shape parameter estimate.

The three distributions defined above are unified under the Generalized Extreme Value distribution expressed by Jenkinson & VonMises(1955). Here the tail index (τ) is related to the shape parameter α as $\tau=1/\alpha$. The tail index in the expression below allows us to discriminate between the three different types of distributions decribed above. In this expression, $\tau=0$ indicates a Gumbel distribution, $\tau<0$ indicates a Frechet distribution and $\tau>0$ implies a Weibull distribution.

$$F_x(x) = e^{-(1+\tau x)^{-\frac{1}{\tau}}}, if\ \tau \neq 0 \qquad \text{(Equation 2)}$$
$$F_x(x) = e^{-e^{-x}}, if\ \tau = 0$$

II.Tail Estimation Methods and the Optimal Tail Size

Extremal theory leads to the generalized extreme value distribution formula that maps onto three types of limiting distributions. In order to discriminate between these it is necessary to estimate the tail index τ . The behavior of extremes in the tails could be determined by directly estimating the three parameters relating to scale (the dispersion of extremal events, a_n), location (the average position of extremes in the distribution, b_n) and the shape of the tail (the density of extremal observations). The literature provides two main approaches to tail index estimation. The first "parametric" school consists of maximum likelihood and regression methods that are used to directly estimate a_n, b_n *and* τ . This method assumes that the extremes are drawn exactly from the limit distribution known as the generalized pareto distribution. Jondeau and Rockinger (2003) use maximum likelihood methods to estimate these parameters across a range of emerging and developed market equity returns. They maximize the following likelihood function across N samples of non overlapping histories drawn from the full period:

$$\mathrm{L}(\mathrm{x}_{\mathrm{i}}, i = 1...., n\tau, a_n, b_n) = \prod_{i=1}^{n} h_{\tau, b_N, a_N}(x_i) I_{\{1+\tau(x-b_{N_i})/a_N>0\}}(m_i) \quad \text{(Equation 3)}$$

where L(.) is the likelihood function maximized over the scale, location and shape parameters.
They find no significant differences in tail indices across emerging or developed markets although they do find some asymmetry in the negative tail of Latin American countries. However there are some serious limitations to this methodology : maximum likelihood methods perform better when tails are thicker providing greater observations exceeding the threshold. However in studying developed markets or relatively short histories of emerging markets, this can be a severe constraint on effective estimation. Further, its success lies on sharp assumptions about the distribution and dependence structure of the data used to calibrate the size of subsample histories and to estimate standard errors. Any weaknesses in these assumptions directly affect the significance of these results.

Jansen and De Vries (1991) show that under a Type II limit law (in the domain of the Frechet distribution) where most of our extreme value distributions operate, maximum likelihood methods are consistent but not the most efficient. A "non-parametric" school offers efficient estimators directly for τ that rely on the largest order statistics of the parent distribution and require only that the data generating distribution is broadly well behaved. In order to meet this criteria all that is usually required is that the data vary regularly at infinity so that for x>0 and $\alpha > 0$ the $\lim_{t\to\infty}(1-F(tx)/1-F(t)) = x^{-\alpha}$. These estimators have a long history, with Hill's index first proposed in 1975.

Hill Estimate: (Hill, 1975)

$$\tau^{H}_{n,m} = \frac{1}{m-1}\sum_{i=1}^{m-1} \ln X_i - \ln X_{n-m,n} \quad \text{where } m > 1 \qquad \text{(Equation 4)}$$

This estimate has been shown to be a consistent estimate for fat tailed distributions by Mason (1982). Consequently the limiting distribution of the $(\alpha_{n,m} - \alpha)\sqrt{m}$ is asymptotically normally distributed $N(0,\alpha^2)$ where m is the largest order statistic in the tail (ie. the cutoff point for the tail). Interestingly consistency is further maintained for a non-independent sequence of the parent variable (X(i)) as long as the dependence is not extremely strong. Intuitively the hill index relies on the step size between extreme observations to extrapolate the behavior of the tails into the broader part of the distribution. In the case that a type II limit law applies, this index is a powerful measure of tail behavior. The primary weakness of this index lies in the need to determine the size of the tail a priori l. In it's first implementation, Du Mouchel (1979) showed a heuristic 10% of the sample size to perform reasonably in large samples. However Pictet, Dacorogna, and Muller (1996) show that in finite samples the expectation of the Hill estimator is biased. Numerous bootstrap and regression methods have also been devised to optimally determine the size of the tail index and Section III discusses one of these methods and its application.

Pickands (1975) proposed an estimator that defines the tail as less than or equal to a quarter of the full sample. This estimator has been shown to be weakly consistent. The strong consistency and asymptotic normality of this estimator has also proved by Dekkers and DeHaan (1993) when the maximum order value m(n) rises rapidly enough relative to sample size (n).

$$\tau^{P}_{n,m} = [\ln(X_m - X_{2m})/(X_{2m} - X_{4m})]/\ln 2 \qquad \text{(Equation 5)}$$

Dekkers and De Haan (1990) also propose an extension to the Hill estimate that takes into account its second moment.

$$\tau_{n,m}^{D} = \tau_{n,m}^{H1} + 1 - \frac{1}{2}(1 - \{\frac{(\tau_{n,m}^{H1})^2}{\tau_{n,m}^{H2}}\})^{-1} \qquad \text{(Equation 6)}$$

where $\tau_{n,m}^{H1}$ is the standard Hill estimate in equation (4) above and

$$\tau_{n,m}^{H2} = \frac{1}{m-1}\sum_{i=1}^{m-1}[\ln X_i - \ln X_m]^2 \qquad \text{(Equation 7)}$$

The authors prove the consistency and normality of $\tau_{n,m}^{D}$.

In all three estimators above, the final tail index estimate relies heavily on the choice of (m). While all three require that m(n) approach infinity in some well-behaved fashion, there is little direction as to how to optimally choose m. The choice of (m) ultimately involves the classic tradeoff between bias and variance. If (m) is chosen conservatively with few order statistics in the tail, then the tail estimate will be sensitive to outliers in the distribution and have a higher variance. On the other hand increasing the tail to extend well into the central part of the distribution creates a more stable index but results in a biased value. This sensitive tradeoff can be dealt with in a variety of ways. A heuristic device employed here in figures 1-3 is to consider the change in the tail estimate for increasing sizes of m. The levels at which the tail index begins to plateau is in some sense the 'optimal' choice of(m)where it is no longer as sensitive to individual observations. Other devices used to make this choice include the quantile – quantile plot and the mean excess function (Gencay, Selcuk 2003).

Figures 1 and 2 in Appendix A show quantile quantile estimation plots for the developed and emerging countries respectively. This is the first diagnostic in determining whether the data have fat tails. QQ plots display the quantiles of the sample data against those of a standard normal distribution. This serves as a useful device to qualitatively compare the normality of the data without any other assumptions. If the data were to be drawn from a normal distribution, then we would expect a linear plot along the 45 degree line. Deviations from linearity are evidence of non-normality. In this case it is particularly telling that the greatest deviation occurs at the tails in each case. The central part of the distribution aligns well with our expectations of a normal distribution. However outside this area the curve in the tails indicates the discrepancy from normality. Furthermore the fact that each of the plots curve out ward indicate greater density in these quantiles of the sample data than in the normal, this is particular evidence of the higher than predicted (according to normal) probability of extreme observations. Finally we note that the emerging markets consistently appear to deviate from normality more strikingly than the developed region. Among developed markets, linearity is maintained for a much larger part of the distribution with curious non-linearities appearing in the far positive and negative tails.

The difference between emerging and developed market tail indices is visually striking. As a first test, I modify a method employed to identify structural changes in the tail index across a series.
In a recent paper Quintos et al (2003) test for structural change in the tail index across a time series of equity returns in the Taiwanese, Thai and Chinese markets. They calculate the tail index on a rolling and recursive basis. The recursive hill estimator $\hat{\alpha}_t$ is estimated from the subsample [1,…..,t]. At each iteration, the window size increases until the subsample encompasses the full sample of the data. The rolling hill estimator $\hat{\alpha}^*{}_t$ is estimated from a fixed subsample size w_t, where at each iteration a single observation is added and one is eliminated in order to maintain the same window size. Rolling this window through the series allows the calculation of a series of tail indices and prevents extreme observations seen in the early part of the sample from influence tail index estimates later in the series. I apply a similar test here on series created by pairwise merging each developed to an emerging return series. This is done simply to see if the recursive or rolling tail index estimated on such a pseudo series reveals any sharp difference in the tail index. Figures 5 and 6 in Appendix A plot the average of recursive and rolling shape parameter estimates ($\hat{\alpha}$ =1/τ) for each developed country across all pairwise combinations with eight emerging countries. Therefore the Australian series in Figure 5 is the average of the eight recursive shape parameter series created by merging Australia to each of the eight emerging markets in the sample.

The average of the recursive shape parameter in Figure 5 is particularly telling, we see that this index across developed countries is typically higher than the second part of the merged series that represents emerging market returns. In all cases, the average tail index declines sharply once the estimator begins to include observations from the emerging market part of the series. We also find that typically the shape parameter is greater than 2 suggesting that at least the first two moments of all series do exist. The rolling estimator (Figure 6) applied to similarly merged series displays similar results, however this is a more volatile series since the window size is held fixed throughout at 300 observations leading to some small sample bias in the estimator calculation.

Our final visual test involves calculating each of the different tail index estimates across a rolling sample of the data. This provides us with information on two fronts, at each level it offers us a comparison across tail index estimation methodologies as well as directing the search for the optimal tail size. As we might expect, the optimal size at which bias and variance tradeoff is minimized varies according to the exact non-parametric method employed. Estimates of the tail index are notoriously sensitive to the size of the tail. The tradeoff is between limiting oneself to 'too small' a tail so that the tail index estimate is highly sensitive to outliers and allowing too many observations from the central distribution into the tail which would bias the tail index estimate upward. We also vary the sample size and choose that size at which the tail estimate appears to plateau out. The graphical results of such a procedure are shown in the Appendix figures 3 and 4. In each pair of figures we present the tail index estimate for the Pickand, Dekkers and Hill estimators respectively and for the developed and emerging markets separately. In table 1 this information is tabulated for sample sizes of 100, 200 and 300 observations. Since we need a precise step size between observations for the non-parameteric measures described above, we limit data from 12th May 1998 onward as this is when the MSCI price indices begin to show daily variation for all countries in the sample.

For emerging and developed markets, the tail index appears to rise until about 200 observations. The hill index starts to stabilize in this zone and then declines. The Dekkers and De Haan estimate tends to keep rising especially in the positive tail. However for both emerging and developed markets, in the negative tail, this estimate starts to stabilize by about 300 observations. The Pickands estimate seems to stabilize quickly but it's asymptotic variance is very large further at small values of k it is extremely volatile. The volatility of the pickands estimate and its asymptotic results lead us to reject this as the method of choice in the rest of the paper. The increase in the Dekkers estimate throughout the positive tail makes it difficult to locate a region of the stability for this estimator. Further since a significant part of the analysis is devoted to comparisons in the tail index across tails we are in need of a method that will consistently perform across tails. This leads us to eliminate Dekkers and De Haan as the tail index for further investigation. Instead we focus on the hill index which has the added advantage of a great deal of attention from the field. As a result there exist various methods to estimate the optimal order size specifically for hill index estimation. The visual plots below suggest the general area of stability for the hill index across regions and tails is at a sample size of between 200-250. For the sake of comparison, we estimate the optimal size from a more traditional econometric technique below. While it is difficult to find a consistent consensus across these many estimates regarding sample size, the general stability around samples of 250 – 300 suggests that this is the appropriate size to consider.

IIA: Weighted Least Squares Method of Optimal Tail Size Selection

One of the major concerns in applying the hill index as the primary metric of tail thickness is the sensitivity of the index to the number of extreme values or order statistics used in the calculation of the index. While it has been shown that the hill index is asymptotically unbiased, it is equally shown to suffer from deep bias in small estimates. There is a well known bias-variance tradeoff - including too few observations reduces the bias but increases the variance of the estimate. On the other hand drawing too many observations from the central part of the distribution causes extreme bias to enter at the cost of reduced variance. One approach offered by Huisman et al (2000) is to calculate the hill index over a range of different tail sizes, then calculate weights over a range of these standard hill indexes where the weights are measured from conventional least squares techniques. This provides an estimate of the hill index, that is a weighted average over various tail indices. The theoretical justification for this method is the following:

The literature presents the bias in the hill index for this class of distributions:

$$F(x) = 1 - \alpha x^{-\alpha}(1 + bx^{-\beta})$$

The impact of the size of the tail index can be seen in the expected value and variance of the hill index at any given size of the tail. In both cases, the bias in the hill index increases with higher k while the variance declines in k. As a result, minimizing bias requires limiting the size of the tail while achieving efficiency requires increasing k. This can be seen in the following equations:

$$E(\gamma(k)) \sim 1/\alpha - b\beta/(\alpha(\alpha + \beta) \qquad \text{(Equation 8)}$$

$$\mathrm{var}(\gamma(k)) \sim 1/k\alpha^2 \qquad \text{(Equation 9)}$$

Given this dilemma Huisman et al offer a method that employs weighted least squares methods to calculate the tail index over a range of choices of k. They find that for small k, the bias term can be approximated by a linear function . In this case equation 9 can be written as

$$(\gamma(k)) = \beta_0 + \beta_i k + \varepsilon_{k,} \qquad k = 1....k \qquad \text{(Equation 10)}$$

This equation offers a way to avoid having to select any one optimal k to determine the estimate of the tail index. Instead the information set of a full range of conventional tail index estimates is used to calculate the tail index of the distribution. In matrix form, this equation can be written $\gamma^* = Z\beta + \varepsilon$ where the γ^* vector consists of $\gamma(k)$ where k ranges from 1 ….K – the range of tail index sizes within which the relationship between the tail index and the number of order statistics is approximately linear. Due to heteroscedasticity in the error term above, the authors propose to use weighted least squares methods where the weights are constructed from the matrix of order statistic sizes. This involves adding a matrix (K * K) matrix with diagonal elements $[\sqrt{1}, \sqrt{2}, \sqrt{3}, ...\sqrt{k}]$ and zeros elsewhere. Solving for the WLS estimate of β results in:

$b_{wls} = (Z'W'WZ)^{-1} Z'W'Wy^*$ - as a result the modified hill estimator is a weighted average of the standard estimators for k=1….k

$$\gamma(k) = \sum_{i=1}^{k} w(k)\gamma(k) \qquad \text{(Equation 11)}$$

where weights are formed from the wls coefficient estimates.

Table 1:Emerging Markets – Weighted Least Squares tail index

Country	Positive	Negative
Argentina	2.44	3.14
Brazil	5.44	4.69
Chile	5.37	3.88
Indonesia	2.86	2.84
Phillipines	3.85	2.81
Korea	4.37	7.80
Taiwan	7.91	5.14
Thailand	3.61	5.59

Table 2:Developed Markets – Weighted Least Squares Tail Index

Country	Positive	Negative
Australia	4.65	4.05
Austria	6.34	4.66
Belgium	3.89	5.33
Canada	5.14	3.60
Germany	4.79	5.49
HongKong	3.82	3.70
Ireland	6.31	3.74
Italy	5.31	5.42
Japan	4.44	6.39
Singapore	4.37	2.84
UK	5.31	4.73
USA	4.19	5.25

Since the aim is to compare tail index estimates, it is important to have a choice of order statistics that is consistent across markets. Du Mouchel's rule of denoting the tail as 10% of the ordered sample is used as a rule of thumb in the literature. The results from the WLS method can be compared to the tail index results calculated from 10% of the tail index size in tables 1 and 2 in the Appendix. We find that the weighted least squares tail index is highly sensitive to the initial size of the tail. This presents a real dilemma to the reliability of this methodology. Further based on the visual plots, we see the estimate calculated here is typically toward the highest end of the rolling sample. This suggests that a large number of center observations are dominating the weighted least square calculation. In some sense this provides us with an upper bound of the tail index estimate. However based on the visual and tabulated results – and the standard use in the literature of DuMouchel's 10% threshold rule, we choose to use this rule on the full sample and use the hill index to estimate the tail index in the bootstrapping and monte-carlo analyses in the following sections. Since our purpose here is to compare the size of the tail index for distributions within a limiting distribution we are able to rely on the broad area of stability in the hill index to calculate and consistently compare the tail behavior across different markets and regions.

III. Asymmetry Across Tails

Tail index estimates range widely across markets. Emerging markets like Argentina, Philippines and Chile have negative tail hill estimates less than 2 (at a constant tail size of 200), while developed markets typically have indexes above 2. Within the same developed region, country estimates of the negative tail index (at tail size of 200 order statistics) vary from 1.77 to 2.58. Given this range the statistical significance of differences in tail estimates must be measured carefully. Related studies typically make distributional assumptions to estimate the tail index and to construct tests of tail index equality. However the range of estimates point to the difficulty of making any universal distributional claims. Therefore we aim to remain agnostic about the implications of the tail index for the underlying distribution and the existence of moments. Although we use a Monte Carlo approach based on the hill index estimate, we also construct a bootstrapping test to identify statistical asymmetry in a way that is free of any distributional assumptions in its construction.

The only work similar to this in nature is by Jondeau and Rockinger (2003). For a set of emerging and developed nations encompassing those studied here, they find almost no significant difference in the tail index either between tails or across markets. Their results relied on the direct estimation of the scale, location and shape parameters of the extreme value distribution. These are estimated by fitting a maximum likelihood function to often sparse extreme observations. They construct likelihood ratio tests to estimate the significance of parameter differences. Jansen and De Vries (1991) exploit the asymptotic normality of the hill index to create comparative test measures distributed χ^2, that are similar to the standard wald statistic. However both of these approaches require a parametrized distribution under the null. This is unfortunate since the exact distribution generating returns continues to be unknown. For this reason we attempt to remain as agnostic as possible. We test our conclusions from any Monte Carlo approach with bootstrapping methods to allow the empirical distribution to speak for itself.

The Monte Carlo approach is based on mapping the tail index to the degrees of freedom of a frechet distribution. Here a word of caution is appropriate - as the tail index can inform us about the existence of moments, it is tempting to use these estimates to draw distributional conclusions. A casual view of hill estimates in the Appendix Table 1 suggests that some moments of the underlying distribution may not exist. However substantial evidence shows that such inferences may not be reliable. Longin (1996) concluded that third and higher moments could be infinite for a century of daily returns on an index of the most traded NYSE stocks. He establishes that while the mean and variance definitely exist, third moments and higher such as skewness and kurtosis could be infinite. Similarly (Wagner and March 2000) show that in small samples, semi to non parametric estimation of the stable return models can fail in small samples. As a result they conclude that while all tail index estimation techniques can be used given the a priori assumption of a Frechet limiting distribution, they cannot be used to infer the underlying distribution from the calculated estimate. Therefore we do not comment here on the existence of moments, and in the Monte Carlo approach we assume we are in the frechet domain of attraction and map the tail index to the degrees of freedom of a student-t distribution.

The data comprise of the Morgan Stanley Capitalization International (MSCI) country price indices from Jan 1st 1993 – May 1st 2003. These indices capture 80% of the market capitalization at each point in time. Local currency equity returns are calculated as the log difference in daily price index observations. Each return series is demeaned and transformed to unit variance to minimize the effect of differing variance in the original data. Since daily prices for some markets are unavailable until 4th May 1998, some of the tests below limit the period to May 5th 1998 – May 1st 2003. We follow our visual test and Du Mouchel's rule using a consistent tail size of 150 order statistics. For the reasons discussed in Section II, we choose to use the hill index and compare tail thicknesses between the positive and negative tail of each market's return distribution.

III.1 Asymmetry Across Tail : Monte Carlo Evidence

Monte carlo methods are used to simulate data from a given distribution and generate confidence intervals against which to compare actual empirical estimates. The previous discussion on EVT indicates that the student-t is an accurate representation of the underlying distribution in cases where the tail index is in the range of [0,2). In order to study the asymmetry in the tails of each return distribution, we simulate a symmetric student-t distribution with degrees of freedom calibrated to each market's actual positive tail index. We rank 10,000 tail index differences and report the 5th and 95th percentile observations. This calibrates the distribution under the null of tail index symmetry. We estimate the statistical significance of the actual tail index difference by comparing it to these confidence intervals.

Among emerging markets, we reject the null of symmetric tails in the cases of Argentina, Chile, Philippines, Taiwan and Thailand. In Table 3 below, the "empirical difference" measures the actual tail index difference calculated as the positive minus the negative tail index, $\tau_{pos} - \tau_{neg} = \tau_{diff.}$ Since the tail index is the negative inverse of the shape parameter, a smaller value indicates a fatter tail. In all statistically significant cases the difference is negative, suggesting that the negative is fatter than the positive tail and consequently market busts in these markets are more probable than booms.

Table 3 : Emerging Market Tail Asymmetry

Country	5^{th} Percentile	95^{th} Percentile	Empirical Difference
Argentina	-0.1476	0.1497	**-0.1701**
Brazil	-0.1801	0.1785	-0.1086
Chile	-0.1524	0.1551	**-0.2546**
Indonesia	-0.1698	0.1643	0.0398
Phillipines	-0.1461	0.1506	**-0.2882**
Korea	-0.1779	0.1823	0.0390
Taiwan	-0.1749	0.1721	**-0.2262**
Thailand	-0.1561	0.1584	**-0.1882**

For developed markets we reject symmetry in Table 4 only in HongKong and Singapore. The common region and sharp difference from other nations in this set suggest that these two countries are more like the emerging markets in their distributional characteristics.

Table 4: Developed Market Tail Asymmetry

Country	5^{th} Percentile	95^{th} Percentile	Empirical Difference
Australia	-0.1980	0.1891	-0.0713
Austria	-0.1813	0.1846	0.1228
Belgium	-0.1948	0.1886	0.0033
Canada	-0.1923	0.1919	-0.0012
Germany	-0.1962	0.1895	-0.0593
Hong Kong	-0.1729	0.1716	**-0.1869**
Ireland	-0.1825	0.1797	-0.0750
Italy	-0.1759	0.1773	-0.0445
Japan	-0.1890	0.1857	-0.1083
Singapore	-0.1832	0.1857	**-0.1933**
UK	-0.1846	0.1860	-0.1127
USA	-0.1825	0.1831	-0.0701

III.2 Asymmetry Across Tails : Wild Bootstrap

The distributional assumptions of the Monte Carlo analysis above depend on the exact calculation of the tail index. However in section II we saw that the hill index is particularly sensitive to the number of order statistics used to define the tail. A qualitative judgment has to be made about the tail size at which the hill plot stabilizes. Further the range of tail index estimates within each country and the differing volatility of the hill plot across markets suggest that the reliability of the mapping between the hill index and the student-t distribution may vary across markets. Therefore we supplement the Monte Carlo results with a non-parametric approach that does not make any distributional assumptions based on the hill index and relies entirely on the empirical distribution.

The wild bootstrap tests for asymmetry by scrambling the actual observations. A sample of the data is multiplied by a randomly selected vector [1,-1] chosen with probability =0.5 to scramble the sign on the observed data. The data is resampled multiple times to render the process completely random. The tail index of the positive and negative tails is then calculated on each of these scrambled series. The tail index difference over these samples yields an estimate of the distribution of tail index difference under the null hypothesis of complete symmetry. We calculate the 5th and 95th percentile observations and compare the empirical tail index difference against these bounds.

The empirical tail index difference varies here from Table 3 above because we use data here from May 12th 1998 onward to take advantage of the daily variation in the MSCI price indices across all markets in the sample. While limiting the study to this latter period does not impact the conclusions of the Monte Carlo approach presented above (results from author), they become relevant in a study that scrambles the signs on daily observations. These results provide an interesting benchmark against which to compare the monte carlo results and particularly to evaluate the impact of the distributional assumptions above.

Table 5 :Wild Bootstrap in Emerging Markets

Country	5th Percentile	95th Percentile	Empirical Difference
Argentina	-0.3869	0.4001	-0.1415
Brazil	-0.4957	0.4912	**-1.1412**
Chile	-0.4683	0.4718	-0.0860
Indonesia	-0.4816	0.4817	-0.2151

Phillipines	-0.4508	0.4479	-0.1742
Korea	-0.5668	0.5657	0.0139
Taiwan	-0.6622	0.6542	-0.1322
Thailand	-0.5133	0.5121	-0.1143

According to this test, we reject the hypothesis of symmetry only in the case of Brazil. However the asymmetry for Brazil is striking. Interestingly Rockinger and Jondeau, using maximum likelihood estimation methods found significant asymmetry only for Brazil. We find that 8 of the 10 largest negative moves in the Brazilian market occur in the late 90s. The fact that the Monte Carlo study did not identify significant asymmetry for Brazil could be because the effect of these observations on the hill index and the associated student-t distribution was diluted when the full period was taken into account. We find similar cases among developed markets below.

Table 6: Wild Bootstrap in Developed Markets

	5^{th} Percentile	95^{th} Percentile	Empirical Difference
Australia	-0.8185	0.8337	-0.3422
Austria	-0.5707	0.5727	0.1038
Belgium	-0.5287	0.5305	**-0.5931**
Canada	-0.5111	0.5019	**-0.5642**
Germany	-0.5558	0.5467	-0.2426
Hong Kong	-0.6154	0.5980	-0.2203
Ireland	-0.4910	0.4974	**-0.5233**
Italy	-0.9942	1.0134	-0.6091
Japan	-0.7430	0.7474	-0.3219
Singapore	-0.5960	0.5866	-0.1236
UK	-0.6045	0.5917	-0.1512
USA	-0.5936	0.6014	-0.4500

This simple bootstrap technique tells an interesting story for developed markets. While the Monte Carlo approach above identified two of the most emerging-like countries in the developed region, in Table 6 above we find significant asymmetry for Belgium, Canada and Ireland. All of these countries had their minimum return in this latter period. This method may be identifying slight deviations from asymmetry while the Monte Carlo technique will only identify asymmetry when it affects the overall tail index calculation and the degrees of freedom on the student-t distribution. This may explain some of the differences between the results of the two methodologies. However despite such a severe test, we are able to identify asymmetry in a few major developed markets.

We find evidence of significant asymmetry between markets in both the emerging and developed region. While the Monte Carlo and bootstrapping approaches reveal evidence of asymmetry in different countries, this could be due to the sensitivity of each approach to extreme observations. The prevalence of asymmetry in the emerging markets relative to the developed countries however suggests that there may be substantial differences in regional tail index estimates. We now turn to systematically estimate these differences and characterize each region's tail behavior.

IV. Regional Differences In Tail Behavior

There is substantial evidence of differing volatilities and levels of efficiency between developed and emerging equity markets. It is plausible that tail indices might be statistically different between the emerging and developed region. Due to the large number of combinations, it is difficult to follow the previous Monte Carlo approach between pairs of markets. Instead we construct bootstrapping tests to estimate the significance of the difference in tail behavior between regions. Bootstrapping methods make no distributional assumptions and are based on repeated sampling from the empirical data. If we can assume that markets in the same region have similar data generating processes, we are able to pool the data and preserve their distributional characteristics. We can then compare a series of "regional returns" with an individual market's returns and estimate the significance of the tail index difference between these series. In the following sections we estimate the difference in tail behavior between markets and their own region and between each market and the alternative region (ie. Chile vs. the developed region). We show evidence of sufficiently similar tail behavior to justify pooling individual countries into regional returns. Based on this, we are able to show that emerging markets have consistently fatter tails than the developed region.

In each of the tests below we bootstrap the original length of the data series from the individual country and respective pool. We calculate the negative tail index and the empirical tail index difference τ_{diff} ($\tau_{market} - \tau_{pool} = \tau_{diff}$) for each sample. This provides the distribution of the null of a common tail index between the region and the market. We report the 5th and 95th percentile of this distribution. We then calculate the empirical negative tail index difference between the market and region. By comparing this test statistic against the percentile bounds of the null distribution we can estimate the statistical significance of the actual difference. Following DuMouchel (1984) rule discussed in Section II, we use 10% of the region's data to calculate the hill index for each region. Once again it is important to note that we are not suggesting that all developed market countries have precisely the same data generating process. However given anecdotal evidence of the regional nature of market crashes and booms, we are led to believe that tail behavior is persistent and preserved under pooling. We choose non-parametric methods in this section so as not to rely on any distributional assumptions that may or may not hold under addition. Our assumption is further supported by the next results on the significance of differences in negative tail behavior between most markets and their associated region.

IV.1. Do Markets in the same region have similar tail behavior?

We first test the assumption of pooling country returns within a similar region. We ask whether individual markets are different from their own pool by comparing each market's negative tail index to that of its own region's pooled return. Consistent rejections of this test would suggest that in pooling the data we are incorrectly combining fundamentally variant data generating processes. However we find that this is not regularly the case. As the results show below, there are only two emerging markets and no developed countries in which the individual market has a statistically different tail index from its region. This supports our methodology and lends confidence to our results of significant differences between emerging and developed markets in their distributional tail behavior.

Table 9: Emerging Markets Vs. Own Pool

Country	5^{th} Percentile	95^{th} Percentile	Empirical Difference
Argentina	-0.4584	0.2959	0.0005
Brazil	-0.5890	0.1172	0.0779
Chile	-0.5920	0.1131	0.1043
Indonesia	-0.3065	0.4364	-0.0142
Phillipines	-0.5224	0.1677	0.0469
Korea	0.3191	1.1652	**-0.1751**
Taiwan	0.0726	1.0385	**-0.1442**
Thailand	-0.1845	0.6496	-0.0580

In Table 9 we compare the negative tail index difference between each market and the emerging region against the null distribution of a common tail index parameter. Each market is part of its own regional pool. We bootstrap repeatedly from this pool and calculate the negative tail index of each sample. The difference between the bootstrapped sample and the market's own negative tail index constructs the null distribution of the same tail behavior between the market and region. The 5th and 95th percentiles of this distribution are reported. As we see, the difference in the negative tail index between the market and the emerging pool ("empirical difference") is statistically significant only for Korea and Taiwan. In both cases the pool has a fatter tail than these individual markets therefore in the rest of the analysis we keep these countries as part of the pooled emerging return. Since the region has a fatter tail than each individual market we are not biasing the negative tail index by keeping these in the sample. On the other hand maintaining the same sample helps compare results across tests in this study. In a similarly constructed test for developed markets, Table 10 shows no evidence of individual markets with different tail behavior than the developed market pool

Table 10: Developed Markets Vs. Own Pool

Country	5th Percentile	95th Percentile	Empirical Difference
Australia	-0.6705	0.2255	0.0480
Austria	-0.5582	0.4392	0.0058
Belgium	-0.3887	0.5623	-0.0127
Canada	-0.2948	0.6180	-0.0218
Germany	-0.0833	0.9566	-0.0606
Hong Kong	-0.7846	0.0855	-0.0831
Ireland	-0.0396	-0.3810	0.6843
Italy	-0.6946	0.2695	0.0492
Japan	-0.4276	0.6175	-0.0172
Singapore	-0.4980	0.4263	0.0044
UK	-0.5338	0.3785	0.0234
USA	-0.1624	0.8263	-0.0618

IV.2. Are there regional differences in tail behavior?

Based on our results above, we accept our prior of tail behavior preserved under pooling. We proceed to test whether emerging markets as a region have systematically fatter tails than the developed market region. We find persistent patterns of differences in the data generating processes between these two broad categories of markets.

Table 11 compares the actual difference between the positive tail indices of each emerging market and the developed regional pool (the "empirical differences", $\tau_{market} - \tau_{pool} = \tau_{diff}$). The bootstrapped simulations correspond to the null hypothesis of a common tail index between each market and the alternative pool. 10,000 samples were bootstrapped from the regional data; the negative tail index difference between each sample and the emerging market's negative tail index were ranked and the 5th and 95th percentiles are reported. We find that five out of the eight emerging markets report statistically different tail indices from the developed market pool. In all cases the rejection occurs on the right hand side of the distribution, suggesting that the emerging market tail index is larger than the developed more than 95% of the time. This indicates slower decay and fatter tails in emerging markets than in the tail of the developed market pool.

Table 11: Individual Emerging Markets Vs. Developed Pool

Country	5th Percentile	95th Percentile	Empirical Difference
Argentina	-0.9890	-0.0915	0.1190
Brazil	-1.1017	-0.2783	0.1964
Chile	-1.1146	-0.2872	0.2228
Indonesia	-0.8328	0.0456	0.1043
Phillipines	-1.0462	-0.2184	0.1654
Korea	-0.2156	0.7669	-0.0566
Taiwan	-0.4545	0.6224	-0.0257
Thailand	-0.6932	0.2388	0.0605

We then ask the converse question and use a similar methodology to test whether developed markets are statistically different in tail behavior from the emerging pool. Table 12 shows seven countries have tail behavior that is significantly different from the emerging region. It is especially striking that the empirical difference between all of these countries and the emerging pool is smaller than the 5th percentile of the null distribution. Once again the distribution is constructed by bootstrapping from the developed region to calculate $\tau_{market} - \tau_{pool} = \tau_{diff}$ (ie. developed market tail - emerging region pooled tail index). In each case, rejections occur on the left hand side of the null distribution suggesting that the emerging tail index is a larger absolute value. The emerging pool therefore exhibits slower decay than any of these individual markets.

Table 12: Individual Developed Markets Vs. Emerging Pool

Country	5th Percentile	95th Percentile	Empirical Difference
Australia	-0.1542	0.6121	-0.0704
Austria	-0.0416	0.8198	-0.1127
Belgium	0.1040	0.9635	-0.1311
Canada	0.2139	1.0257	-0.1413
Germany	0.4214	1.3642	-0.1791
Hong Kong	-0.2677	0.4775	-0.0354
Ireland	0.1173	1.0882	-0.1580
Italy	-0.1962	0.6695	-0.0693
Japan	0.0936	1.0143	-0.1357
Singapore	0.0193	0.8304	0.1141
UK	-0.0152	0.7814	-0.0951
USA	0.3381	1.2282	-0.1803

The results here are internally consistent and suggest that emerging markets do indeed have systematically fatter tails than developed markets. These features in the data are consistent with several theories including those of weaker efficiency in these markets and the higher probability of herding by asymmetrically informed foreign investors.

Under this methodology the results are strongly consistent and supportive of systematically fatter tails in emerging markets either seen individually or as a region. This finding suggests that risk calculations that depend on tail behavior must be done with that extreme caution when applying the same threshold rules to markets across these regions

IV:CONCLUSIONS

Extreme value theory offers exciting possibilities to further our understanding of tail events. While existing literature has applied EVT to risk management and to improve standard statistical measures of dispersion – there has been little work done to understand differences in tail behavior between crashes and booms or between markets in different regions . Many of these applications are sensitive to the particular market and direction to which they are being applied – as we have seen here statistically significant differences exist in the tail index both between sides of the distribution and between emerging and developed markets. From the perspective of the tail index – each emerging market is sufficiently similar to its region to allow us to pool return data and create a regional series. This strong universality within the region might help to explain the geographic nature of market crashes that characterized the Latin American and Asian crises. These results provide much direction for further work, possible extensions would be to account for dependence in the data and test the regional results under some form of persistence. Finally this can then be applied to measures of value at risk that take the particular tail index into consideration. While current studies start to extend VAR in this way, we need to explicitly calibrate the tail behavior of each market and allow for differing levels of tail thickness between markets. As we have shown here, markets between regions have variant tail behavior. In the area of risk management for emerging equities, an accurate measure and understanding of market specific tail behavior is critical to hedging risk in these markets.

Appendix

Figure 1

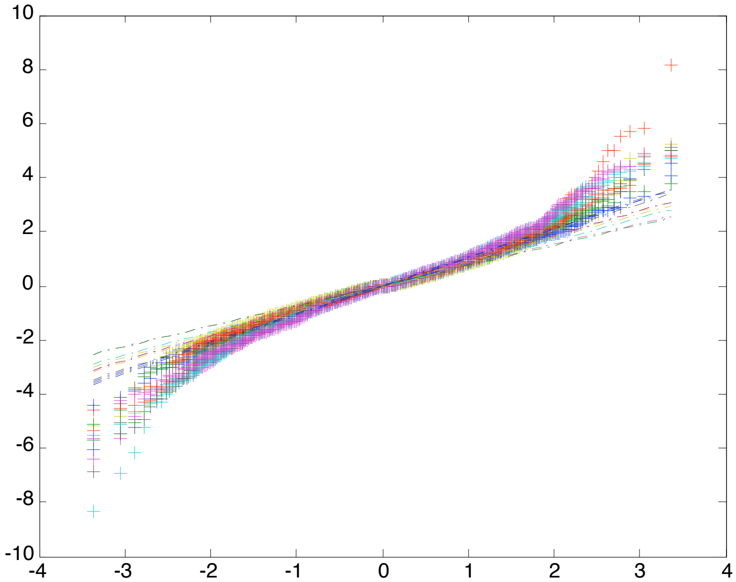

Figure 2

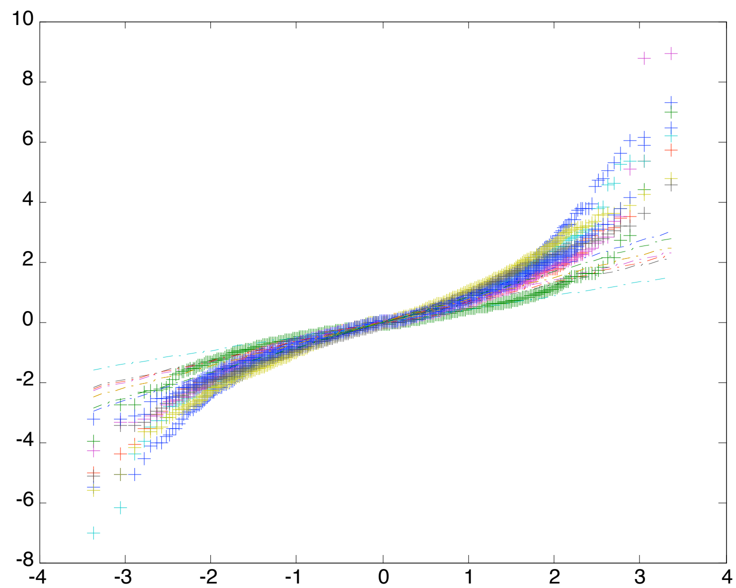

Table 1A-1B. Emerging Market Tail Estimates

Positive	Pickand= $\frac{1}{\tau^{p}}$			Hill= $\frac{1}{\tau^{H}}$			Dekkers & DeHaan= $\frac{1}{\tau^{D}}$		
	M=100	M=200	M=300	M=100	M=200	M=300	M=100	M=200	M=300
Argentina	-0.1900	0.2131	2.4774	1.8741	1.4607	1.2141	2.287	11.38	4.26
Brazil	-0.0025	0.2582	3.3469	1.1254	1.2760	1.1660	2.2089	1.8238	2.2435
Chile	0.0330	0.0917	2.6847	1.7015	1.5260	1.3833	2.8776	4.924	7.1983
Indonesia	-0.0266	0.3153	2.8278	1.9396	1.5429	1.2879	2.1295	4.4404	5.9077
Phillipines	0.2017	0.0929	2.0360	1.7714	1.5642	1.2460	2.4471	3.973	10.74
Korea	-0.3273	0.3475	3.5613	2.4883	1.9235	1.4478	1.768	2.1035	2.8843
Taiwan	-0.1241	-0.0885	1.4786	2.3807	1.7267	1.3984	1.8022	2.6083	4.6506
Thailand	0.1621	0.0024	2.0159	1.8496	1.6576	1.2487	2.2447	3.0838	8.8625

M=Number of order statistics. The sample of (n) observations is first sorted in ascending order, observations from 1 to the M^{th} smallest observation are used in each tail index calculation.

Negative	Pickand= $\frac{1}{\tau^{p}}$			Hill= $\frac{1}{\tau^{H}}$			Dekkers & DeHaan= $\frac{1}{\tau^{D}}$		
	M=100	M=200	M=300	M=100	M=200	M=300	M=100	M=200	M=300
Argentina	0.1900	0.2131	2.4474	2.0156	1.7002	1.4129	2.0652	2.7863	4.9656
Brazil	-0.0025	0.2582	3.3469	2.2656	1.9988	1.7140	1.8486	2.0155	2.2911
Chile	0.0330	0.0917	2.6847	1.7846	1.7205	1.4638	2.4853	2.5885	3.2697
Indonesia	-0.0266	0.3153	2.8278	2.1547	1.8059	1.4025	1.8882	2.3209	3.6790
Phillipines	0.2017	0.0929	2.0360	1.9456	1.7082	1.4245	2.1790	2.6426	4.2292
Korea	-0.3273	0.3473	3.5613	2.4744	2.0981	1.5271	1.7703	1.9394	2.3822
Taiwan	-0.1241	-0.0885	1.4786	2.5129	2.1461	1.6884	1.7671	1.8955	2.0439
Thailand	0.1621	0.0024	2.0159	1.9639	1.7013	1.4483	2.0535	2.9592	3.8013

*M=Number of order statistics. The sample of (n) observations is first sorted in ascending order, observations from 1 to the M^{th} smallest observation are used in each tail index calculation.

Table 2: Developed Markets Tail Index

Negative	Pickand= $\frac{1}{\tau^{p}}$			Hill= $\frac{1}{\tau^{H}}$			Dekkers & DeHaan = $\frac{1}{\tau^{D}}$		
	M=100	M=200	M=300	M=100	M=200	M=300	M=100	M=200	M=300
Australia	-0.3669	0.2354	3.0963	2.3903	2.1942	1.7310	1.7947	1.8630	2.0635
Austria	-0.1698	0.5255	3.2816	1.9566	1.7754	1.3857	2/0516	2.4872	5.0162
Belgium	-0.2329	0.4065	2.5977	2.7838	2.1444	1.4565	1.7295	1.9065	2.7907
Canada	-0.0378	0.1780	3.2194	2.6891	2.0828	1.6402	1.7415	1.9366	2.1068
Germany	-0.1568	0.2033	2.8750	2.6919	2.1126	1.6480	1.7370	1.9240	2.1934
HongKong	0.0131	0.0880	2.7544	2.2674	1.8988	1.5279	1.8393	2.1261	2.4759
Ireland	0.0677	0.2066	3.3163	2.5880	2.0080	1.5802	1.7698	2.0470	2.2572
Italy	-0.2175	0.2317	2.4174	2.5912	2.1338	1.6256	1.7493	2.9008	2.2566
Japan	-0.5650	0.3948	2.2587	2.6939	2.3158	1.5769	1.7362	1.8273	2.5949
Singapore	0.0329	-0.0200	2.3265	2.4565	1.9932	1.6520	1.7294	2.0121	2.3939
UK	-0.1858	0.1439	2.6319	2.6511	2.1239	1.6449	1.7548	1.9034	2.2119
US	-0.6495	0.4516	2.6796	3.1148	2.5735	1.6294	1.7110	1.7534	2.0083

*M=Number of order statistics. The sample of (n) observations is first sorted in ascending order, observations from 1 to the M^{th} smallest observation are used in each tail index calculation.

Positive	Pickand= $\frac{1}{\tau^{p}}$			Hill= $\frac{1}{\tau^{H}}$			Dekkers & DeHaan= $\frac{1}{\tau^{D}}$		
	M=100	M=200	M=300	M=100	M=200	M=300	M=100	M=200	M=300
Australia	-0.3669	0.2354	3.0963	2.0481	1.9022	1.6276	1.9601	2.1221	2.2740
Austria	-0.1698	0.5255	3.2816	2.0604	1.6262	1.3491	1.9602	3.1606	6.1994
Belgium	-0.2329	0.4065	2.5977	2.1907	1.6081	1.3584	1.8962	3.7008	5.0741
Canada	-0.0378	0.1780	3.2194	2.1248	1.727	1.5123	1.9060	2.5560	2.6259
Germany	-0.1568	0.2033	2.8750	2.4493	2.0801	1.5522	1.7813	1.9322	2.5799

HongKong	0.0131	0.0880	2.7544	2.0471	1.7155	1.4727	2.0316	2.6967	3.1742
Ireland	0.0677	0.2066	3.3163	2.0617	1.7807	1.4856	2.0181	2.4434	3.3206
Italy	-0.2175	0.2317	2.4174	1.9821	1.8127	1.4185	2.0207	2.2950	3.8316
Japan	-0.5650	0.3948	2.2587	2.3720	1.9836	1.4779	1.8029	2.0258	2.9327
Singapore	0.0329	-0.0200	2.3265	2.3329	1.8731	1.4553	1.8074	2.1770	2.7087
UK	-0.1858	0.1439	2.6319	2.8023	2.0045	1.5908	1.7270	2.0023	2.7833
US	-0.6495	0.4516	2.6796	2.6648	1.9352	1.5324	1.7482	2.0862	2.6152

*M=Number of order statistics. The sample of (n) observations is first sorted in ascending order, observations from 1 to the M^{th} smallest observation are used in each tail index calculation.

Emerging Market Autocorrelation

Argentina	Brazil	Chile	Indonesia	Phillipines	Korea	Taiwan	Thailand
0.0600	0.0074	0.1011	0.0525	0.0449	0.0200	0.0099	0.0774
-0.0032	-0.0067	0.0344	0.0112	-0.0170	-0.0207	0.0216	0.0216
-0.0422	-0.0092	0.0189	0.0090	-0.0187	-0.0181	0.0218	-0.0135
-0.0027	-0.0149	0.0268	0.0037	-0.0178	-0.0175	-0.0316	-0.0207
0.0170	-0.0092	0.0520	-0.0043	0.0024	-0.0251	0.0187	0.0052

Developed Markets Autocorrelation Lags 1: 5

	Australia	Austria	Belgium	Canada	Germany	HongKong	Ireland	Italy
1	-0.0092	0.0245	0.1089	0.0066	-0.001	-0.001	0.069	0.0047
2	-0.0144	-0.0148	-0.0122	-0.0145	-0.0201	-0.0182	-0.0162	0.0018
3	0.0196	0.0123	0.073	0.0096	-0.0208	0.0112	0.0091	-0.0235
4	-0.0219	0.0447	-0.0101	-0.0419	0.0257	-0.0035	0.0187	0.0304
5	-0.0121	-0.0162	-0.0455	-0.0196	-0.0175	-0.023	-0.0111	-0.0265

	Japan	Singapore	UK	USA
1	0.0062	0.0482	-0.0003	-0.013
2	-0.0085	-0.0026	-0.0515	-0.0142
3	-0.0138	0.0091	-0.0731	-0.0264
4	-0.0156	0.0116	-0.0266	-0.0115
5	-0.0197	-0.0002	-0.0331	-0.0147

Figure 3

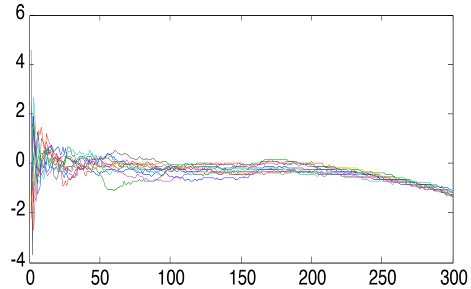

Figure 4

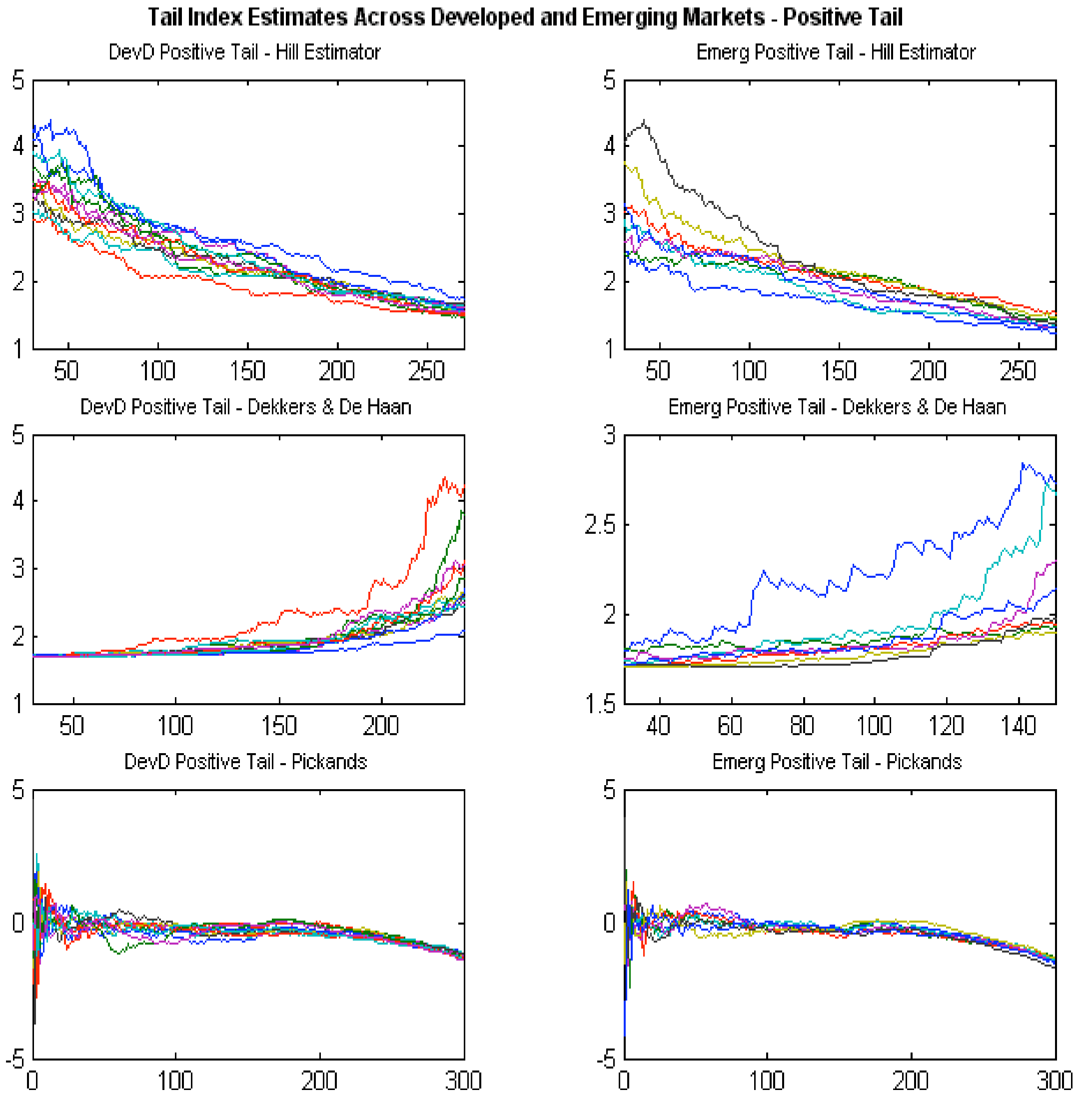
Tail Index Estimates Across Developed and Emerging Markets - Positive Tail
DevD Positive Tail - Hill Estimator
Emerg Positive Tail - Hill Estimator
DevD Positive Tail - Dekkers & De Haan
Emerg Positive Tail - Dekkers & De Haan
DevD Positive Tail - Pickands
Emerg Positive Tail - Pickands

Figure 5 – Average Recursive Tail Index Across Emerging Countries

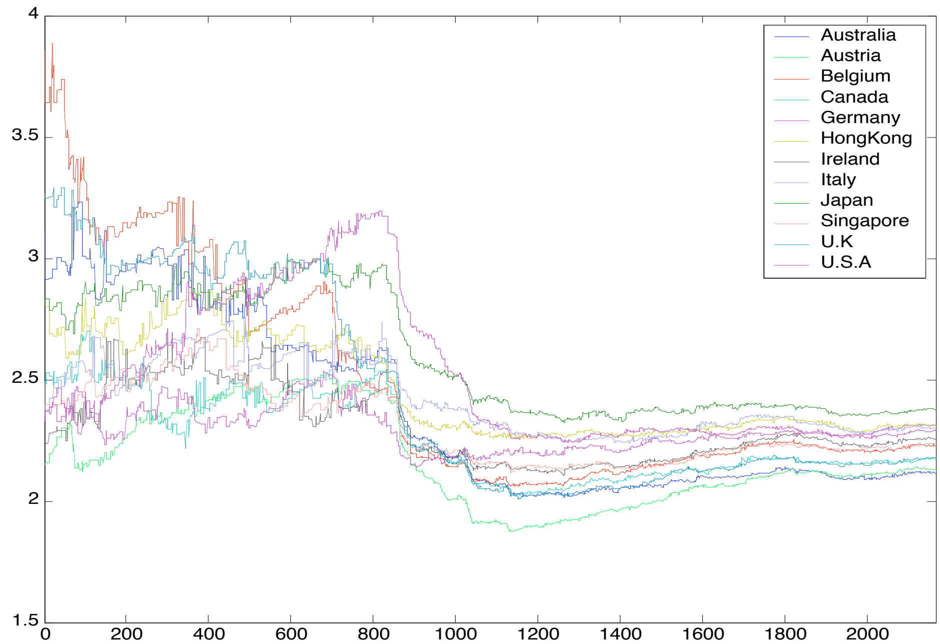

Figure 6 – Average Rolling Tail Index Across Emerging Countries